

# Structure analysis of mutually incommensurate composite crystal $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$

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## Abstract

Single-crystal X-ray structure analysis of mutually incommensurate  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” has been performed by the composite approach which leads to average substructures and their relative arrangement. The composite crystal structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  has the  $\text{CuO}_2$  substructure and the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure. The  $\text{CuO}_2$  substructure with  $a_1 = 10.598(2) \text{ \AA}$ ,  $b = 6.189(2) \text{ \AA}$ ,  $c_1 = 2.825(2) \text{ \AA}$ ,  $\beta_1 = 90.19(4)^\circ$ ,  $V_1 = 185.4(1) \text{ \AA}^3$ ,  $Z = 4$  and space group  $F2/m$  has the plane of edge-shared one-dimensional  $\text{CuO}_2$  chains along the  $c$ -axis. The  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure with  $a_2 = 10.629(2) \text{ \AA}$ ,  $b = 6.189(2) \text{ \AA}$ ,  $c_2 = 3.517(1) \text{ \AA}$ ,  $\beta_2 = 94.36(3)^\circ$ ,  $V_2 = 230.7(1) \text{ \AA}^3$ ,  $Z = 4$  and space group  $F2/m$  forms the sheet of (Ca, Y) atoms in the  $ac$ -plane. By considering  $(3 + 1)$ -dimensional superspace group symmetry, it is concluded that the incommensurate composite crystal structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  should be described by the combination of  $F2/m$  for the  $\text{CuO}_2$  substructure and  $F2/c$  for the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure. The composite approach has made clear that the plane of  $\text{CuO}_2$  chains and the sheet of (Ca, Y) atoms stack alternately to form a mutually incommensurate composite crystal with layered substructures.

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## 1. Introduction

Recently, many cuprates with quasi-one-dimensional  $\text{CuO}_2$  chain have attracted much attention because low-dimensional Heisenberg antiferromagnets with the  $S = 1/2$  spins enhance quantum fluctuations in the spin liquid state [1–4]. This phenomenon seems to be related to the occur-

rence of superconductivity in low-dimensional electron systems. It is known that some compounds of them form composite crystals with mutually incommensurate substructures. In the  $(\text{Sr}_{2-x}(\text{Ca}, \text{La}, \text{Y})_x\text{Cu}_2\text{O}_3)_{0.7+\delta}\text{CuO}_2$ , “ $\text{Sr}_{14-x}(\text{Ca}, \text{La}, \text{Y})_x\text{Cu}_{24}\text{O}_{41}$ ” series, the  $\text{CuO}_2$  chain in the  $\text{CuO}_2$  substructure shows unique structural modulations and acts as hole reservoir of the two-legged  $\text{Cu}_2\text{O}_3$  ladder in the  $\text{Sr}_{2-x}(\text{Ca}, \text{La}, \text{Y})_x\text{Cu}_2\text{O}_3$  substructure [5]. By the use of polycrystalline sample [6], on the other hand, it has been preliminarily revealed that so-called  $\text{Ca}_{2+x}\text{Y}_{2-x}\text{Cu}_5\text{O}_{10}$  series [7]

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also form composite crystal structures with the  $\text{CuO}_2$  substructure and the  $\text{Ca}_{0.5+x}\text{Y}_{0.5-x}$  one and that it is expressed as  $(\text{Ca}_{0.5+x}\text{Y}_{0.5-x})_{0.8+\delta}\text{CuO}_2$ .

In many compounds containing  $\text{CuO}_2$  chains,  $(\text{Sr}_{2-x}(\text{Ca}, \text{La}, \text{Y})_x\text{Cu}_2\text{O}_3)_{0.7+\delta}\text{CuO}_2$  and  $(\text{Ca}_{0.5+x}\text{Y}_{0.5-x})_{0.8+\delta}\text{CuO}_2$  are especially interesting because the amount of holes in the  $\text{CuO}_2$  chain can be easily controlled by doping the trivalent atoms. According to the amount of holes in the  $\text{CuO}_2$  chain, they show a variety of magnetic features with very complicated spin arrangements. To understand their magnetic properties, it is profitable to clarify the mutually incommensurate composite structure by using single-crystalline sample.

In the present study, single-crystal X-ray structure analysis of mutually incommensurate  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.8+\delta}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” has been performed by the composite approach which leads to average substructures and their relative arrangement [8].

## 2. Experimental

Single composite crystals of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.8+\delta}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” were grown by the traveling solvent floating zone (TSFZ) method [9,10]. A single composite crystal of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.8+\delta}\text{CuO}_2$  with dimensions about  $0.2\text{ mm} \times 0.2\text{ mm} \times 0.05\text{ mm}$  was used for the structure analysis. The composite diffraction patterns of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.8+\delta}\text{CuO}_2$  were identified by X-ray precession method with Mo  $\text{K}\alpha$  radiation ( $\lambda = 0.71073\text{ \AA}$ ). In the procedure of the camera technique, imaging plate (BAS UR 5 in.  $\times$  5 in. type, Fuji Photo Film Co. Ltd.) was used to detect the weak satellite reflections of composite structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.8+\delta}\text{CuO}_2$ . We have taken the  $c$ -axis as the mutually incommensurate direction in accord with the structure determination of  $(\text{Sr}_2\text{Cu}_2\text{O}_3)_{0.70}\text{CuO}_2$ , “ $\text{Sr}_{14}\text{Cu}_{24}\text{O}_{41}$ ” in the  $(\text{Sr}_{2-x}\text{Ca}_x\text{Cu}_2\text{O}_3)_{0.7+\delta}\text{CuO}_2$  series [5]. The X-ray diffraction data of both substructures were collected together with common reflections at room temperature using Rigaku AFC5 diffractometer (Mo  $\text{K}\alpha$  radiation). The lattice parameters and data collection process are summarized in Table 1. The  $\delta$  in the formula of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.8+\delta}\text{CuO}_2$  was calculated as  $(c_1/c_2) - 0.8 = 0.003$ . All the calculations for the structure refinement of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  were carried out using the FMLS system [11]. The mutually incommensurate composite structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  was drawn by the use of PRJMS in the REMOS system [12].

## 3. Results and discussion

By means of X-ray diffraction method, we have observed the composite diffraction pattern of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” with the  $\text{CuO}_2$  substructure and the second  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure (Fig. 1). In the present study, it was proved that a minimal reciprocal set  $(a^*, b^*, c_1^*, c_2^*)$  forms the monoclinic quasi-lattice of the composite crystal structure

Table 1  
Experimental summary for  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ”

Substructure	$\text{CuO}_2$	$\text{Ca}_{0.5}\text{Y}_{0.5}$
Crystal system	Monoclinic	Monoclinic
$a$ ( $\text{\AA}$ )	10.598(2)	10.629(2)
$b$ ( $\text{\AA}$ )	6.189(2)	6.189(2)
$c$ ( $\text{\AA}$ )	2.825(2)	3.517(1)
$\beta$ ( $^\circ$ )	90.19(4)	94.36(3)
$Z$	4	4
Diffractometer	Rigaku AFC-5	
Radiation ( $\text{\AA}$ )	Mo $\text{K}\alpha$ 0.71073	
$\theta$ range: min, max ( $^\circ$ )	1.5, 45.0	
Scan mode	$2\theta - \omega$	
Data set	$h$ : 0–21 $k$ : 0–12 $l$ : –6–6	$h$ : 0–21 $k$ : 0–12 $l$ : –7–7
Unique reflections	414	512

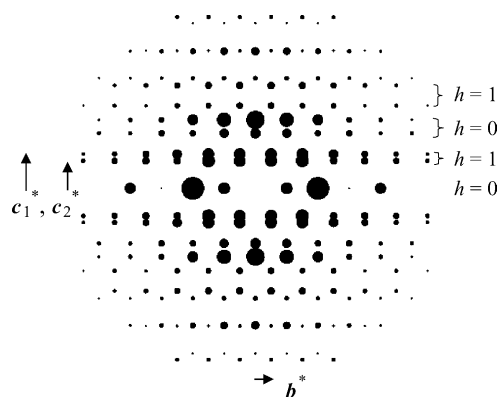


Fig. 1. The schematic drawing of the composite diffraction pattern of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” with  $0klm$  and  $1klm$ -type X-ray reflections.

of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  and  $c_2^* = (a^*, b^*, c_1^*)\sigma(\alpha\ 0\ \gamma)^T$ . This shows that the mutually incommensurate composite structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  obeys the symmetry of  $(3+1)$ -dimensional superspace group [11,12]. As illustrated in Fig. 2, the composite crystal structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  is commensurate along the direction normal to the  $c$ -axis because the orthogonal projections  $a_1'$  of  $a_1$  and  $a_2'$  of  $a_2$  are equivalent together. The mutual incommensurability between the average substructures are expressed as  $\sigma(\alpha\ 0\ \gamma)$ ,

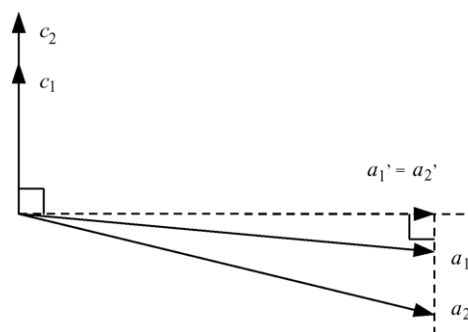


Fig. 2. The mutually incommensurate in-plane relation between the  $\text{CuO}_2$  substructure and the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure in  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ .

where  $\alpha = [\sin(\beta_2 - 90) - \sin(\beta_1 - 90)](a_2/c_2) = 0.219$  and  $\gamma = c_1/c_2 = 0.803$ .

The average substructure of the  $\text{CuO}_2$ , defined by the monoclinic space group  $F2/m$ , has been determined with the edge-shared one-dimensional  $\text{CuO}_2$  chains running along the  $c$ -axis. Because the reflections with  $l=0$  are common to both substructures, we have excluded these reflections to refine each substructure. The final refinement converged with  $R$ -value of 0.057 and  $R_w$ -value of 0.067 using 361 reflections excluding  $hk0$  type. The  $\text{CuO}_2$  substructure is essentially isostructural with that of  $\text{Ca}_{0.83}\text{CuO}_2$ , which was preliminarily investigated using the polycrystalline sample [13,14].

The average substructure of the  $\text{Ca}_{0.5}\text{Y}_{0.5}$ , defined by the monoclinic space group  $F2/m$ , has been determined with the sheet of (Ca, Y) atoms in the  $ac$ -plane. Because no superstructure reflections that suggest the ordering of Ca or Y atoms have been observed, we have employed the structure model with Ca and Y statistically distributed in the sheet of (Ca, Y) atoms. The final refinement converged with  $R$ -value of 0.113 and  $R_w$ -value of 0.152 using 450 reflections excluding  $hk0$  type. The rather high values of  $R$  and  $R_w$  suggest that Ca and Y atoms form the positionally modulated structure that will be discussed in our future studies. All of the atomic parameters of each substructure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  are listed in Table 2.

The common  $hk0$  reflections are essential to show clearly that  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” is a composite crystal with the  $\text{CuO}_2$  substructure and the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure. By considering  $(3+1)$ -dimensional superspace group symmetry, the reflection condition for  $h0l$  of the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure as  $l=2n$  indicates that the origin of the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  should be shifted by  $(1/4\ 1/4\ 0)$ . This means that the space group of the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  should be converted into  $F2/c$ . The refinement of the commensurate section smoothly converged to an  $R$ -value of 0.082 and an  $R_w$ -value of 0.092 with 47 common reflections. The corresponding atomic parameters are listed in Table 3. By the composite approach in the present study, we have successfully demonstrated that  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” forms a composite crystal with two interpenetrating substructures. The mutually incommensurate composite crystal structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$  is drawn in Fig. 3.

Table 2

Atomic parameters and equivalent temperature factors ( $\text{\AA}^2$ ) of the  $\text{CuO}_2$  substructure and the  $\text{Ca}_{0.5}\text{Y}_{0.5}$  substructure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ”

	Occupancy	$x$	$y$	$z$	$B_{\text{eq}}$
Cu	1.0	0.0	0.0	0.0	0.68(2)
O	1.0	0.6242(5)	0.0	0.001(2)	4.9(3)
Ca	0.48(2)	0.0	0.0	0.0	0.78(2)
Y	0.52	0.0	0.0	0.0	1.0(1)

The estimated standard deviations are given in parentheses. In each substructure analysis, all of the atomic parameters are refined by monoclinic  $F2/m$  symmetry.

Table 3

Atomic parameters and isotropic temperature factors ( $\text{\AA}^2$ ) of the composite crystal structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” projected along the mutually incommensurate  $c_1$ - and  $c_2$ -axes

	Occupancy	$x$	$y$	$z$	$B_{\text{iso}}$
Cu	1.0	0.0	0.0	–	0.35(9)
O	1.0	0.626(4)	0.0	–	4.5(11)
Ca	0.52(3)	0.25	0.25	–	0.6(9)
Y	0.48	0.25	0.25	–	0.3(4)

The estimated standard deviations are given in parentheses. The occupancy of Y atom, Occ.(Y), is constrained as Occ.(Y) = 1 – Occ.(Ca).

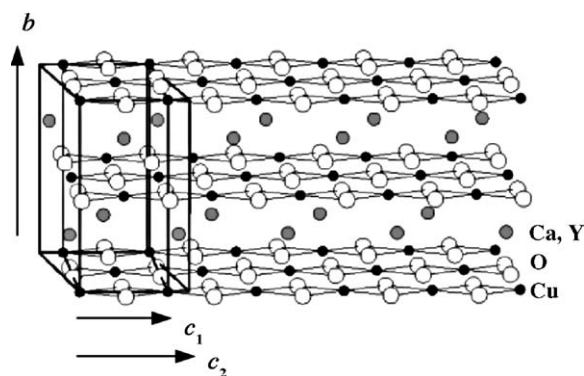


Fig. 3. The perspective view of the mutually incommensurate composite crystal structure of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ .

In the present structure analysis, it is confirmed that the amount of doped Y atom is equivalent to that of Ca atom in our sample. From the chemical composition of  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , therefore, it is expected that the average valence of Cu atom is +2.0 in the  $\text{CuO}_2$  chain. With the results in Table 2, in fact, the calculated Cu–O distances and bond–valence sum [15] of Cu atom in the  $\text{CuO}_2$  chain are 1.931(5)  $\text{\AA}$  and 2.02, respectively. The agreement between the formal valence and the bond–valence sum indicates that Y-doping certainly control the valence of Cu atom in the  $(\text{Ca}_{0.5+x}\text{Y}_{0.5-x})_{0.8+\delta}\text{CuO}_2$ , “ $\text{Ca}_{2+x}\text{Y}_{2-x}\text{Cu}_5\text{O}_{10}$ ” series. Since holes are not doped in the average substructure of the  $\text{CuO}_2$  chain in  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , accordingly, magnetic interaction between nearest-neighbor  $\text{Cu}^{2+}$  ions are possible in the  $\text{CuO}_2$  chain. Mizuno et al. theoretically predicted that the exchange interaction between the nearest-neighbor  $\text{Cu}^{2+}$  ions in the edge-shared  $\text{CuO}_2$  chain turns out to be ferromagnetic if Cu–O–Cu angle is below  $95^\circ$  [16]. In the average  $\text{CuO}_2$  chain in  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , the Cu–O–Cu angle is  $94.1(2)^\circ$ . Evidently, our results obtained by the composite approach well explain the ferromagnetic feature in the  $\text{CuO}_2$  chain in  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ , “ $\text{Ca}_2\text{Y}_2\text{Cu}_5\text{O}_{10}$ ” [7].

#### 4. Conclusions

In the present study, single-crystal X-ray structure analysis of mutually incommensurate  $(\text{Ca}_{0.5}\text{Y}_{0.5})_{0.80}\text{CuO}_2$ ,

“Ca<sub>2</sub>Y<sub>2</sub>Cu<sub>5</sub>O<sub>10</sub>” has been performed by the composite approach which leads to average substructures and their relative arrangement. The average substructures of the CuO<sub>2</sub> and the Ca<sub>0.5</sub>Y<sub>0.5</sub> have been determined with the plane of edge-shared one-dimensional CuO<sub>2</sub> chains running along the *c*-axis and the sheet of (Ca, Y) atoms in the *ac*-plane, respectively. By considering (3 + 1)-dimensional superspace group symmetry, it is concluded that the mutually incommensurate composite crystal structure of (Ca<sub>0.5</sub>Y<sub>0.5</sub>)<sub>0.80</sub>CuO<sub>2</sub> should be described by the combination of monoclinic *F2/m* for the CuO<sub>2</sub> substructure and monoclinic *F2/c* for the Ca<sub>0.5</sub>Y<sub>0.5</sub> substructure. The composite approach has made clear that the plane of CuO<sub>2</sub> chains and the sheet of (Ca, Y) atoms stack alternately to form a mutually incommensurate composite crystal. According to the results on the chemical composition of (Ca<sub>0.5</sub>Y<sub>0.5</sub>)<sub>0.80</sub>CuO<sub>2</sub> and the bond–valence sum calculation of Cu atom in the CuO<sub>2</sub> chain, it is confirmed that Y-doping certainly control the valence of Cu atom in the (Ca<sub>0.5+x</sub>Y<sub>0.5-x</sub>)<sub>0.8+δ</sub>CuO<sub>2</sub>, “Ca<sub>2+x</sub>Y<sub>2-x</sub>Cu<sub>5</sub>O<sub>10</sub>” series. The ferromagnetic feature in the CuO<sub>2</sub> chain in (Ca<sub>0.5</sub>Y<sub>0.5</sub>)<sub>0.80</sub>CuO<sub>2</sub>, “Ca<sub>2</sub>Y<sub>2</sub>Cu<sub>5</sub>O<sub>10</sub>” has been well explained by considering the Cu–O–Cu angle in the average CuO<sub>2</sub> chain.

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